PHGN 100 Equation Sheet

Useful Constants

$g = 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$, $G = 6.67 \times 10^{-11}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{kg}}$

Vector Principles

$\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath} + V_z\hat{k}$

$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$

Kinematics

$\vec{r}_{\mathrm{cm}} = \frac{1}{m_{\mathrm{tot}}}\sum_j m_j\vec{r}_j$

$\vec{r}(t) = \vec{r}(t_{\mathrm{i}}) + \int_{t_{\mathrm{i}}}^{t}\vec{v}(t')\mathrm{d}t'$

$\vec{v}(t) = \frac{\mathrm{d}\vec{r}(t)}{\mathrm{d}t} = \vec{v}(t_{\mathrm{i}}) + \int_{t_{\mathrm{i}}}^{t}\vec{a}(t')\mathrm{d}t'$

$\vec{a}(t) = \frac{\mathrm{d}\vec{v}(t)}{\mathrm{d}t}$

$\vec{v}_{\mathrm{av}}\,\Big|_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} = \frac{1}{t_{\mathrm{f}}-t_{\mathrm{i}}}\int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}}\vec{v}(t)\mathrm{d}t = \frac{\vec{r}(t_{\mathrm{f}})-\vec{r}(t_{\mathrm{i}})}{t_{\mathrm{f}}-t_{\mathrm{i}}}$

$\vec{a}_{\mathrm{av}}\,\Big|_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} = \frac{1}{t_{\mathrm{f}}-t_{\mathrm{i}}}\int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}}\vec{a}(t)\mathrm{d}t = \frac{\vec{v}(t_{\mathrm{f}})-\vec{v}(t_{\mathrm{i}})}{t_{\mathrm{f}}-t_{\mathrm{i}}}$

Translational Dynamics

$$\sum_j \vec{F}_{j\mathrm{b}} = m_{\mathrm{b}}\vec{a}_{\mathrm{b,cm}}$$

$\vec{F}_{\mathrm{ab}} = -\vec{F}_{\mathrm{ba}}$

Weight force: $W_{\mathrm{Eb}} = m_{\mathrm{b}}g$

Friction: $f_{\mathrm{s,ab}} \leq \mu_{\mathrm{s}}N_{\mathrm{ab}}$; $f_{\mathrm{k,ab}} = \mu_{\mathrm{k}}N_{\mathrm{ab}}$

Ideal Spring: $F_{\mathrm{sp\ ab},x}(t) = -k[x(t) - x_0]$

Circular Motion Kinematics

$s(t)$ = Arc Length

$\theta(t) \equiv \frac{s(t)}{R} = \theta(t_{\mathrm{i}}) + \int_{t_{\mathrm{i}}}^{t}\omega(t')\mathrm{d}t'$

$\omega(t) \equiv \frac{\mathrm{d}\theta(t)}{\mathrm{d}t} = \omega(t_{\mathrm{i}}) + \int_{t_{\mathrm{i}}}^{t}\alpha(t')\mathrm{d}t'$

$\alpha(t) \equiv \frac{\mathrm{d}\omega(t)}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2}$

$a_{\mathrm{c}}(t) = \frac{v_{\mathrm{t}}(t)^2}{R} = \omega(t)^2 R$

$v_{\mathrm{t}}(t) \equiv \frac{\mathrm{d}s(t)}{\mathrm{d}t} = \omega(t)R$

$a_{\mathrm{t}}(t) \equiv \frac{\mathrm{d}v_{\mathrm{t}}(t)}{\mathrm{d}t} = \alpha(t)R$

Simple Harmonic Motion

Position: $x(t) = A\sin(\omega t + \delta) + x_{\mathrm{eq}}$

Relationship between $a_x(t)$ and $x(t)$:

$a_x(t) = -\omega^2\left[x(t) - x_{\mathrm{eq}}\right]$

Mass-Spring System: $\omega = \sqrt{\frac{k}{m}}$

Frequency, Angular Frequency, Period:

$f = \frac{1}{T}$; $\omega = 2\pi f$

Rotational Dynamics

$$\sum_j \vec{\tau}_{j\mathrm{b}} = I_{\mathrm{b}}\vec{\alpha}_{\mathrm{b}}$$

Torque: $\vec{\tau}_{\mathrm{ab}} = \vec{r} \times \vec{F}_{\mathrm{ab}}$

$\tau_{\mathrm{ab}} = r_{\perp}F_{\mathrm{ab}} = r\,F_{\mathrm{ab}\perp} = r\,F_{\mathrm{ab}}\sin\theta$

Rolling without Slipping:

$x_{\mathrm{cm}} = R\theta$; $v_{\mathrm{cm}} = R\omega$; $a_{\mathrm{cm}} = R\alpha$

Moments of Inertia

Point Mass: $I = mr^2$

Cylindrical Shell/Hoop: $I_{\mathrm{cm}} = mR^2$

Thin Rod: $I_{\mathrm{cm}} = \frac{1}{12}mL^2$; $I_{\mathrm{end}} = \frac{1}{3}mL^2$

Solid Disk/Cylinder $I_{\mathrm{cm}} = \frac{1}{2}mR^2$

Solid Sphere: $I_{\mathrm{cm}} = \frac{2}{5}mR^2$

Hollow Sphere: $I_{\mathrm{cm}} = \frac{2}{3}mR^2$

Work and Energy

$W_{\text{tot}} = \Delta E_{\text{sys}}$

Work: $W_{\vec{F}_{ab}} = \int_{\vec{s}_{\text{b,i}}}^{\vec{s}_{\text{b,f}}} \vec{F}_{\text{ab}} \cdot d\vec{s}$

$d\vec{s} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$

$W_{\vec{\tau}_{\text{ab}}} = \int_{\theta_{\text{b,i}}}^{\theta_{\text{b,f}}} \tau_{\text{ab}} \mathrm{d}\theta$

Kinetic Energies:

$$K_{\text{T}} = \frac{1}{2} m v_{\text{cm}}^2;\ K_{\text{R}} = \frac{1}{2} I \omega_{\text{cm}}^2$$

Thermal Energy: $\Delta E_{\text{therm,ab}} = f_{\text{k,ab}} s_{\text{ab}}$

Change in PE: $\Delta U_{\vec{F}_{\text{ab}}} \equiv -\int_{\vec{s}_{\text{b,i}}}^{\vec{s}_{\text{b,f}}} \vec{F}_{\text{ab}} \cdot \mathrm{d}\vec{s}$

Earth's Gravitational PE:

$$\Delta U_G = -\frac{G m_{\text{E}} m_{\text{b}}}{r_{\text{b,f}}} + \frac{G m_{\text{E}} m_{\text{b}}}{r_{\text{b,i}}}$$

Near Earth: $\Delta U_g = m_b g \left(y_{\text{b,f}} - y_{\text{b,i}}\right)$

Ideal Spring PE:

$$\Delta U_{\text{sp}} = \frac{1}{2} k \left(x_{\text{b,f}} - x_0\right)^2 - \frac{1}{2} k \left(x_{\text{b,i}} - x_0\right)^2$$

Impulse and Momenta

$$\sum_j \vec{I}_{\vec{F}_{\text{ab}}} = \Delta \vec{p}_{\text{b}}$$

$$\sum_j \vec{I}_{\vec{\tau}_{\text{ab}}} = \Delta \vec{L}_{\text{b}}$$

Linear Momentum: $\vec{p} \equiv m \vec{v}_{\text{cm}}$

Angular Momentum: $\vec{L}_{\text{T}} \equiv \vec{r} \times \vec{p};\ \vec{L}_{\text{R}} = I\vec{\omega}$

Impulse: $\vec{I}_{\vec{F}_{\text{ab}}} \equiv \int_{t_{\text{i}}}^{t_{\text{f}}} \vec{F}_{\text{ab}}\, \mathrm{d}t;\ \vec{I}_{\vec{\tau}_{\text{ab}}} \equiv \int_{t_{\text{i}}}^{t_{\text{f}}} \vec{\tau}_{\text{ab}}\, \mathrm{d}t;$

Average Force and Linear Momentum:

$$\sum_j \vec{F}_{j\text{b,av}} = \frac{\Delta \vec{p}_b}{\Delta t}$$

Average Torque and Angular Momentum:

$$\sum_j \vec{\tau}_{j\text{b,av}} = \frac{\Delta \vec{L}_b}{\Delta t}$$

Kinetic Energies and Momenta:

$K_{\text{T}} = \frac{p^2}{2m};\ K_{\text{R}} = \frac{L^2}{2I}$

Elastic collisions, 2 objects, 1D:

$v_{2\text{f},x} - v_{1\text{f},x} = v_{1\text{i},x} - v_{2\text{i},x}$

Potential Energy Functions

Force from PE:

$$\vec{F}_{\text{ab}} = -\vec{\nabla} U_{\vec{F}_{\text{ab}}};\text{in 1D } F_{\text{ab}} = -\frac{\mathrm{d}U_{\vec{F}_{\text{ab}}}}{\mathrm{d}x}$$

Inertial Calculus

$m_{\text{tot}} = \int \mathrm{d}m;\ I_{\text{tot}} = \int \mathrm{d}I$

$\vec{r}_{\text{cm}} = \frac{1}{m_{\text{tot}}} \int \vec{r}_{dm} \mathrm{d}m$

$\mathrm{d}m = \rho \mathrm{d}V$ (3D); $\mathrm{d}m = \sigma \mathrm{dA}$ (2D); $\mathrm{d}m = \lambda \mathrm{d}\ell$ (1D)

Advanced Gravitation

$\vec{F}_{\text{G},j\text{b}} = -\sum_j \frac{G m_j m_{\text{b}}}{r_{j\text{b}}^2} \hat{r}_{j\text{b}} = -\sum_j \frac{G m_j m_{\text{b}}}{r_{j\text{b}}^3} \vec{r}_{j\text{b}}$

$$\vec{g}(\vec{r}_{\text{b}}) = -\sum_j \frac{G m_j}{r_{j\text{b}}^2} \hat{r}_{j\text{b}} = -\sum_j \frac{G m_j}{r_{j\text{b}}^3} \vec{r}_{j\text{b}}$$