Physics 200: Fundamental Equations

Maxwell's Equations

Gauss's Law for Electric Field: $\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \frac{Q_{\text{encl}}}{\varepsilon_0} = 4\pi k Q_{\text{encl}}$; Electric Flux: $\Phi_E = \int \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$
Gauss's Law for Magnetic Field: $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}} = 0$; Magnetic Flux: $\Phi_B = \int \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{A}}$
Ampère/Maxwell: $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \mu_0 I_{\text{thru}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$
Faraday's Law: $\mathcal{E}_{\text{induced}} = \oint (\vec{\boldsymbol{E}} + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}) \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\Phi_B}{dt}$

Fields, Forces, and Energy

Electric field: $d\vec{\boldsymbol{E}} = \frac{k\,dQ}{r^2}\hat{\boldsymbol{r}} = \frac{k\,dQ}{r^3}\vec{\boldsymbol{r}}$; $\vec{\boldsymbol{F}}^{\text{elec}}_{\text{on q}} = q\vec{\boldsymbol{E}}_{\text{at q}}$;
Electric potential: $\Delta V = V_B - V_A = -\int_A^B \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}}$; $E_x = -\frac{dV}{dx}$; $dV = \frac{k\,dQ}{r}$
Electrostatic energy: $\Delta U_{\text{of q}} = q\,\Delta V$
Dielectrics: $\epsilon_{\text{in}} = \kappa_E \epsilon_0$; $C = \kappa_E C_0$
Magnetic field: $d\vec{\boldsymbol{B}} = \frac{\mu_0 I\, d\vec{\boldsymbol{\ell}} \times \hat{\boldsymbol{r}}}{4\pi r^2} = \frac{\mu_0 I\, d\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{r}}}{4\pi r^3}$;
Magnetic force: $d\vec{\boldsymbol{F}} = I\, d\vec{\boldsymbol{\ell}} \times \vec{\boldsymbol{B}}$; $\vec{\boldsymbol{F}}^{\text{mag}}_{\text{on q}} = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$
Magnetic dipole: $\vec{\boldsymbol{\mu}} = NI\vec{\boldsymbol{A}}$; $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$; $U = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}}$

Circuits

Resistors: $V = IR$; $dR = \frac{\rho\, dL}{A}$; $R_{series} = \sum_i R_i$; $R_{parallel} = \left(\sum R_i^{-1}\right)^{-1}$
Capacitors: $C = \frac{Q}{|\Delta V|}$; $U_{\text{C}} = \frac{1}{2}C(\Delta V)^2$; $C_{series} = \left(\sum_i C_i^{-1}\right)^{-1}$; $C_{\text{parallel}} = \sum_i C_i$; $C = \kappa_E C_0$
Current: $I = \frac{dQ}{dt} = n|q|v_{\text{d}}A$; $\vec{\boldsymbol{J}} = nq\vec{\boldsymbol{v}}_{\text{d}}$
Power: $P = I\,\Delta V$
Kirchhoff's Laws: $\sum_{\text{Closed loop}} \Delta V_i = 0$; $\sum I_{\text{in}} = \sum I_{\text{out}}$
RC Circuits: $Q(t) = Q_{\text{final}}\left(1 - e^{-t/RC}\right)$; $Q(t) = Q_{\text{initial}} e^{-t/RC}$
AC Circuits: $X_{\text{C}} = \frac{1}{\omega C}$; $V_{\text{C}} = IX_{\text{C}}$; $Z = \sqrt{R^2 + X_{\text{C}}^2}$; $V = IZ$; $V_{rms} = V_{peak}/\sqrt{2}$
Inductors: $\mathcal{E}_{\text{ind}} = -L\frac{dI}{dt}$; $L = N\Phi_{\text{B, 1 turn}}/I$; $U_{\text{B}} = \frac{1}{2}LI^2$
Inductance: $M_{12} = N_2\Phi_{\text{B, one turn of 2}}/I_{\text{in 1}}$ $\quad \mathcal{E}_1 = -M_{12}\frac{dI_2}{dt}$
LR Circuits: $I(t) = I_{\text{final}}\left(1 - e^{-t/(L/R)}\right)$; $I(t) = I_{\text{initial}} e^{-t/(L/R)}$

Electromagnetic Waves Optics and Field Energy Density

Field Energy/Momentum: $u_E = \frac{1}{2}\varepsilon_0 E^2$; $u_B = \frac{1}{2\mu_0}B^2$; $p = U/c$
Wave Properties: $v = \lambda f$; $k = 2\pi/\lambda$; $\omega = 2\pi f$; $E = cB$
Intensity: $\vec{\boldsymbol{S}} = \frac{1}{\mu_0}\left(\vec{\boldsymbol{E}} \times \vec{\boldsymbol{B}}\right)$; $I = |\vec{\boldsymbol{S}}|_{avg} = c\frac{1}{2}\varepsilon_0 E_{\text{m}}^2 = \frac{P_{\text{av}}}{A}$
Reflection/Refraction: $v_{\text{in material}} = c/n_1$; $\theta_{\text{inc}} = \theta_{\text{ref}}$; $n_1 \sin(\theta_{\text{inc}}) = n_2 \sin(\theta_{\text{trans}})$
Interference: Constructive: $\Delta r = m\lambda$; Destructive: $\Delta r = (m + 1/2)\lambda$
Bragg: $2d\sin(\theta) = m\lambda$; Double-slit: $d\sin(\theta) = m\lambda$ or $dy_m/R = m\lambda$ for small θ

Additional Information/Useful Constants

Electric fields: $E_{\text{inf sheet}} = \frac{\sigma}{2\varepsilon_0}$; $E_{\text{inf line}} = 2k\lambda/r$; $\vec{\boldsymbol{E}}_{\text{charged ring}} = \frac{kQx}{(x^2+a^2)^{3/2}}\hat{\boldsymbol{\imath}}$; $C_{\text{parallel plate}} = \frac{\varepsilon_0 A}{d}$
Magnetic fields: $B_{\text{infinite wire}} = \frac{\mu_0 I}{2\pi r}$; $B_{\text{solenoid}} = \mu_0 nI$; $L_{\text{solenoid}} = \mu_0 n^2 A\ell$; $B_{\text{current loop}} = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}$
Fundamental Charge: $e = 1.602 \times 10^{-19}$ C; Electron Mass: $m_{\text{e}} = 9.11 \times 10^{-31}$ kg
Proton Mass: $m_{\text{p}} = 1.673 \times 10^{-27}$ kg;
$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9(\text{N}\cdot\text{m}^2/\text{C}^2)$ $\quad \epsilon_0 = 8.85 \times 10^{-12}\frac{\text{F}}{\text{m}}$; $\mu_0 = 4\pi \times 10^{-7}\frac{\text{Tm}}{\text{A}} = 1.2566 \times 10^{-6}\frac{\text{Tm}}{\text{A}}$;
$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3 \times 10^8$ m/s;