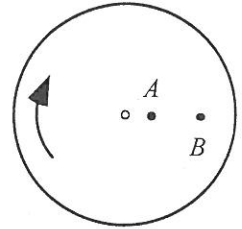Rotational Mechanics Problems

1. A disk with a moment of inertia of 0.2 kg·m² rotates at 300 revolutions per minute. It took 40 s for the disk to reach that rotation rate starting from rest. Consider a point A on the disk that is 1 cm from the axis of rotation and another point B that is farther from this axis. Ten seconds after starting from rest:

Disk rotating
clockwise -- top view

a) Will the magnitude of the angular acceleration of point A be (a) *greater than*, (b) *less than*, or (c) *equal to* the magnitude of the angular acceleration of point B?
   Explain.   connected   so   same α

b) Will the magnitude of the angular velocity of the point A be (a) *greater than*, (b) *less than*, or (c) *equal to* the magnitude of the angular velocity of point B?
   Explain.   connected   so   same Δθ   in same Δt

c) Will the magnitude of the linear velocity of the point A be (a) *greater than*, (b) *less than*, or (c) *equal to* the magnitude of the linear velocity of point B?
   Explain.   A travels less distance in same time

d) What was the angular acceleration of the disk?
$$\omega_f = 300 \frac{rev}{min} \left(\frac{1 min}{60 s}\right)\left(\frac{2\pi rad}{1 rev}\right) = 10\pi \; rad/s$$
$$\omega_i = 0$$
$$t = 40 sec$$
$$\alpha = \frac{\Delta \omega}{t} = \frac{10\pi}{40} = \boxed{\frac{\pi}{4} \; rad/s^2}$$

e) What force was necessary to create that angular acceleration if the force was applied at point A?
$$\tau = I\alpha = .2\left(\frac{\pi}{4}\right) = \frac{\pi}{20} = Fr = F(.01)$$
$$\boxed{F = 5\pi \; N}$$

2. The second hand on a wall clock is 20 cm long. Calculate:
   a. The angular/rotational speed of the second hand.
   $$T = 60 sec \qquad \omega = \frac{2\pi}{60} = \boxed{\frac{\pi}{30} \; rad/s}$$

   b. The linear speed of the tip of the second hand.
   $$v = \omega r = \left(\frac{\pi}{30}\right)(.2) = \boxed{\frac{\pi}{150} \; m/s}$$

   c. The angular/rotational acceleration of the second hand.
   $$0 \quad b/c \quad constant \; speed.$$

3. A fly sits on a potter's wheel 0.3 m from its axle. The wheel's rotational speed decreases from 4 rad/s to 2 rad/s in 5 seconds. Determine:
   a. The wheel's average angular acceleration.

   $$\alpha = \frac{\Delta \omega}{t} = \frac{2-4}{5} = \boxed{-.4 \ rad/s}$$

   b. The angle through which the fly turns during the 5 seconds.

   $$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$
   $$= 0 + 4(5) + \frac{1}{2}(-.4)(5^2) = \boxed{15 \ rad}$$

   c. The distance traveled by the fly during the 5 seconds.

   $$\Delta x = \Delta \theta \ r = (15 \ rad)(.3) = \boxed{4.5 \ m}$$

4. There are certain types of stars called Neutron stars. These are created sometimes during Supernova explosions where the forces are so strong, they push protons into electrons and the result is a creation of many, many neutrons. Because neutrons do not repel each other, they can be packed into super dense material. A single thimble of neutron star matter would weigh 100 million tons on earth!!
   a. One such Neutron star, PSR 0329+54, has been discovered to have a diameter of about 30 km. And it seems to be spinning once every 0.714 seconds. What is it angular velocity?

   $$\omega = \frac{2\pi}{T} = \frac{2\pi}{.714} = 8.8 \ rad/s$$

   b. If you were to stand on the surface of the star at the equator, besides being crushed by the ENORMOUS gravitational forces, what would be your translational speed?

   $$V = \omega r = (8.8)(30 \ km) = 264 \ km/s \ or \ \boxed{264,000 \ m/s}$$

5. The equatorial diameter of the Earth is 12,756 km. Castle Rock, CO sits at a latitude of 39.3722°N which means that if you were to draw lines from the center of the Earth to Castle Rock and the point of the equator directly south from us, it would form an angle of 39.3722°. With that idea, calculate the translational speed that we are going as we revolve around the axis of the Earth.

   $$C = 2\pi(R\cos\theta)$$
   $$= 2\pi\left(\frac{12756}{2}\cos(39.3722)\right)$$
   $$= 30,979 \ km$$
   $$V = \frac{C}{T} = \frac{30,979,000}{24 \cdot 60 \cdot 60} = \boxed{359 \ m/s \ !!!}$$