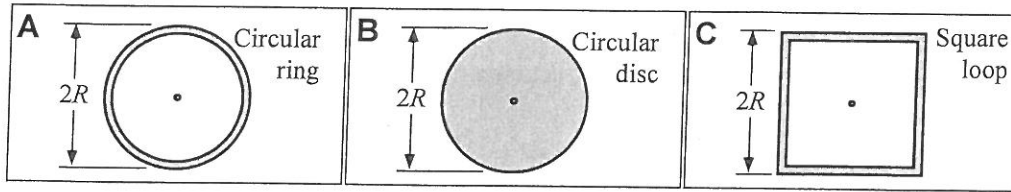


Torque and Inertia Problems

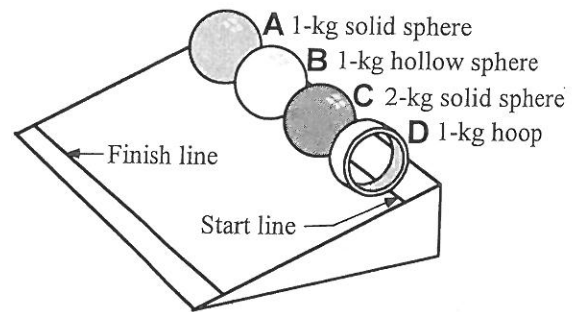
1. Three flat objects (circular ring, circular disc, and square loop) have the same mass M and the same outer dimension (circular objects have diameters of $2R$ and the square loop has sides of $2R$). The small circle at the center of each figure represents the axis of rotation for these objects. This axis of rotation passes through the center of mass and is perpendicular to the plane of the objects.



Rank the moment of inertia of these objects about this axis of rotation.

C	A	B	OR	<input type="checkbox"/>	<input type="checkbox"/>
1	2	3		All	Cannot
Greatest		Least		the same	determine

2. Four objects are placed in a row at the same height near the top of a ramp, and are released from rest at the same time. The objects are (A) a 1-kg solid sphere; (B) a 1-kg hollow sphere; (C) a 2-kg solid sphere; and (D) a 1-kg thin hoop. All four objects have the same diameter, and the hoop has a width that is one-quarter its diameter. The time it takes the objects to reach the finish line near the bottom of the ramp is recorded. The moment of inertia for a solid sphere is $\frac{2}{5}MR^2$; for a hollow sphere it is $\frac{2}{3}MR^2$; and for a hoop it is MR^2 .



Rank the four objects from fastest (shortest time) down the ramp to slowest.

Fastest 1 A 2 B 3 C 4 D Slowest

OR, The times are all the same for these objects.

OR, We cannot determine the ranking for the times for these objects.

Please explain your reasoning.

$A: I = \frac{2}{5}(1)R^2 = \frac{2}{5}R^2$
 $B: I = \frac{2}{3}(1)R^2 = \frac{2}{3}R^2$
 $C: I = \frac{2}{5}(2)R^2 = \frac{4}{5}R^2$
 $D: I = 1R^2 = R^2$

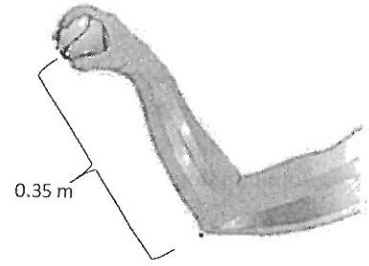
3. A thin cord is wrapped around a non-uniform grindstone of radius 0.3 m supported by bearings that produce negligible friction torque. The cord exerts a steady 20 N tension force on the grindstone, causing it to accelerate from rest to 60 rad/s in 12 seconds. Determine the rotational inertia of the grindstone. (1.2 kgm²)

$$\tau = Fr = 20(0.3) = 6 \text{ Nm}$$

$$\alpha = \frac{\Delta\omega}{t} = \frac{60 - 0}{12} = 5 \text{ rad/s}^2$$

$$I = \frac{\tau}{\alpha} = \frac{6}{5} = 1.2 \text{ kgm}^2$$

4. A baseball player loosening up his arm before a game tosses a 0.15 kg baseball, using only the rotation of his forearm to accelerate the ball. The forearm has a mass of 1.5 kg and the length from the elbow to the ball's center is 35 cm. The ball starts at rest and is released with a speed of 30 m/s in 0.3 seconds.



- a. Find the constant angular acceleration of the arm and ball. (285.7 rad/s²)

$$v_f = 30 \text{ m/s} \quad \omega_f = \frac{v_f}{r} = \frac{30}{0.35} = 85.7 \text{ rad/s}$$

$$\alpha = \frac{85.7 - 0}{0.3} = 285.7 \text{ rad/s}^2$$

- b. Calculate the moment of inertia of the arm and ball system. (Hint: The ball is an object moving around an axis and the forearm is a rod rotating around an axis on one end) (0.0796 kgm²)

$$I_{\text{tot}} = I_{\text{ball}} + I_{\text{arm}} \\ = m_b r^2 + \frac{1}{3} m_{\text{arm}} L^2 = (0.15)(0.35)^2 + \frac{1}{3}(1.5)(0.35)^2 = 0.0796 \text{ kgm}^2$$

- c. Find the torque exerted on the system during the process of throwing. (22.75 Nm)

$$\tau = I\alpha = (0.0796)(285.7) = 22.75 \text{ Nm}$$

5. A potter's wheel having a radius of 0.5 m and a moment of inertia of 12 kgm² is rotation freely at 50 rev/min. The potter can stop the wheel in 6.0 sec by pressing a wet rag against the rim and exerting a radially inward force of 70 N. Find the effective coefficient of kinetic friction between the wheel and wet rag. (0.299)

$$\omega_f = 0$$

$$\omega_i = \frac{50 \text{ rev}}{\text{min}} \left(\frac{1 \text{ min}}{60 \text{ sec}} \right) \left(\frac{2\pi}{1 \text{ rev}} \right) \\ = 5.26 \frac{\text{rad}}{\text{s}}$$

$$t = 6.0 \text{ sec}$$

$$\alpha = \frac{\Delta\omega}{t} = -0.873 \text{ rad/s}^2$$

$$\tau = I\alpha = 12(-0.873) \\ = -10.47 \text{ Nm}$$

$$\tau = F \cdot r = f \cdot r = \mu F_N \cdot r$$

↑
force slowing
it down is
friction

$$\mu = \frac{\tau}{F_N \cdot r} = \frac{10.47}{70(0.5)}$$

$$= 0.299$$

6. A 150 kg merry-go-round in the shape of a uniform, solid, horizontal disk of radius 1.5 m is set in motion by wrapping a rope around the supporting pole with a diameter of 0.4 m. If the maximum tension force on the rope is 400 N and the rope has a total length of 7 m long, what is the maximum speed the merry-go-round can achieve in rpm? (116 rpm)

$$I = \frac{1}{2} MR^2 \\ = \frac{1}{2} (150)(1.5)^2 \\ = 168.75 \text{ kgm}^2$$

$$\tau = F \cdot r = 400 \left(\frac{0.4}{2} \right) \\ = 80 \text{ Nm}$$

$$\alpha = \frac{\tau}{I} = \frac{80}{168.75} = 0.474 \text{ rad/s}^2$$

circumference of inner pole = 4π

$$\frac{7 \text{ m}}{4\pi} = \# \text{ of revs} = 5.57 \text{ rev} \therefore \theta_f = 35 \text{ rad}$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta) \\ = 0^2 + 2(0.474)(35)$$

$$\omega_f = 12.15 \frac{\text{rad}}{\text{s}} \left(\frac{60 \text{ s}}{1 \text{ min}} \right) \left(\frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 116 \text{ rpm}$$