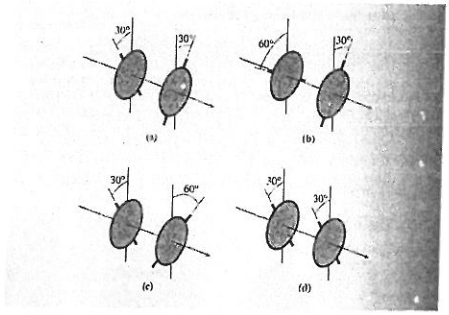
Polarization Problems

1. Unpolarized light of equal intensity is incident on four pairs of polarizing filters. Rank in order, from largest to smallest, the intensities I_a to I_d transmitted through the second polarizer of each pair.



D, A, B = C

2. Two polarizers are oriented at 60° to one another. Unpolarized light is sent through them. What fraction of light intensity is transmitted?

I_0 | I_1 | I_2

$I_2 = I_1 \cos^2(60) = \frac{1}{2} I_0 \cos^2(60)$

$= \boxed{\frac{1}{8} I_0}$

3. What angle should the axes of two polarizers be placed so as to reduce the intensity of the light for 1/3 the original level?

$\frac{1}{3} = \frac{1}{2}\cos^2(\theta)$

$\frac{2}{3} = \cos^2\theta$

$\cos^{-1}\left(\sqrt{\frac{2}{3}}\right) = \boxed{35.26°}$

4. A 200mW horizontally polarized laser beam passes through a polarizing filter whose axis is 25° from the vertical. What is the power of the laser beam as it emerges from the filter?

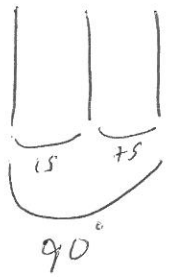
$200\cos^2(65) = \boxed{35.7\text{mW}}$

5. What angle should the axes of two polarizers be placed so as to reduce the intensity of the light for 1/10 the original level?

$\frac{1}{10} = \frac{1}{2}\cos^2\theta$

$\cos^{-1}\left(\sqrt{\frac{1}{5}}\right) = \boxed{63.4°}$

6. If 2 polarizers a placed at 90° to each other no light gets through. But what happens if you place another polarizer at 15° from the 1st polarizer? What is the resultant intensity factor?

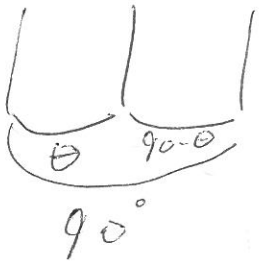


$\frac{1}{2}\cos^2(15)\cos^2(75) = .03125 = \frac{1}{32}$

7. Unpolarized light of intensity I_o is incident on a stack of 7 polarizers, each with its axis rotated 15° clockwise with respect to the previous filter. What light intensity emerges from the last polarizer?

$\frac{1}{2}\left(\cos^2(15)\right)^6 = .33 I_o$

8. Two polarizers are 90° off from each-other. At what angle do I need to place a third polarizer so that the resultant intensity is 1/12 the original?
Hint: cos(a-b) = cos(a)cos(b) + sin(a)sin(b) and sin(2a) = 2sin(a)cos(a)



$\frac{1}{12} = \frac{1}{2}\cos^2(\theta)\cos^2(90-\theta)$

$\frac{1}{6} = \cos^2\theta\left(\cos(90)\cos\theta + \sin 90 \sin\theta\right)^2$

$= \cos^2\theta \sin^2\theta \cdot = \left(\cos\theta\sin\theta\right)^2$

$\frac{1}{6} = \left(\frac{\sin 2\theta}{2}\right)^2$

$\frac{2}{\sqrt{6}} = \sin 2\theta$

$\theta = 27.4°$